DYNAMIC RESPONSE OF SURFACE AND EMBEDDED RECTANGULAR FOUNDATIONS FOR BODY AND SURFACE WAVE EXCITATIONS

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SYNOPSIS

A class of important soil-structure interaction phenomena, namely, wave scattering (diffraction), is introduced and accounted for by a simple concept of filtering. The spectra of dynamic responses of rigid, massless surface and embedded rectangular foundations welded to an elastic half-space and subjected to generally obliquely incident body (SH, SV, and P) waves and horizontally propagating surface (Rayleigh) waves are presented as functions of an equivalent dimensionless frequency parameter for various angles of incidence. The results indicate that both the directivity (angle of incidence) and the type (SH, SV, P, or Rayleigh) of incident seismic wave have a marked effect on the nature and magnitude of the foundation response. Torsional response is maximum for horizontally propagating SH waves, while rocking response becomes predominant for horizontally propagating SV and Rayleigh waves for rectangular foundations with usual shallow ratio of embedment depth to lateral width. In all cases, the translational components of response undergo marked decreases, with the increase in dimensionless frequency parameter, from the corresponding free-field motions. The results are also presented to investigate the influence of the embedment-lateral width ratio on the dynamic response of the foundations; the translational motions decrease and the rotational motions increase with the increase in this ratio. The future research need for adaptation of the filtering concept to seismic excitations is discussed. The accuracy of the results predicted by the filtering concept are compared with those obtained by numerical solution of associated mixed boundary value problems. Finally, the cost of computation by the present concept for the practical range of interest of the dimensionless frequency parameter is only a fraction (~ 1:1000) of that needed to solve the associated mixed boundary value problem.

RESUME

L'idée de filtrage nous sert comme outil pour caractériser le phénomène de diffraction des ondes dans les problèmes d'interaction sol-structure. On présente le spectre du comportement d'une fondation rectangulaire à poids négligeable soit retranchée ou bien à la surface. Les ondes de surfaces Rayleigh et les ondes volumétriques de dilatation et de cisaillement vertical et horizontal sont étudiées. Le tout est présenté en fonction de l'angle d'incidence de l'onde et d'un paramètre non-dimentionnel de fréquences. Les calculs nécessaires sont de l'ordre de 1/1000 comparés à ceux utilisés pour solutionner des équations associées à des conditions de bornes mixtes.

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INTRODUCTION

In the seismic analysis of structures with rigid foundation, inputs simulating earthquake excitation are specified in three mutually perpendicular directions as translational ground motions that are the same over the entire foundation area. The primary assumption underlying such practice is that all particles of the ground in contact with the foundation oscillate in phase and with the same amplitude everywhere. This is the case if the near-surface seismic excitation consists only of vertically propagating plane body waves; also, the longer the apparent wave length of oblique plane waves with respect to the foundation dimension, the more realistic the assumption becomes. For structures with rigid embedded foundations, however, the practice of specifying input by translational motions only can be quite unsatisfactory even for long-period vertically propagating plane body waves. Nor is it unusual that important, if not dominant, contributions to strong motion are made by obliquely incident body waves and by surface waves. Furthermore, studies of earthquake damage to horizontally extended structures (typical of nuclear power plants) and investigations of the recorded accelerograms indicate that the amplitude of the motion of the building foundation is generally less than that of the free-field surface motion and that the shorter the wave length or the higher the frequency of oscillation, the greater the difference in amplitude (1).

In the general case of an obliquely incident plane wave, the particle motions of the media will not be in phase at the media-foundation interface. In the event of an appreciable contrast in impedance between the media and the foundation, coupled with the condition that the apparent wavelength of the impinging wave is comparable with the foundation dimensions, there arises an important class of soil-structure interaction effects -- wave scattering. These effects are manifested by the attenuation of high-frequency components in the translational motion of the foundation, along with torsional and rocking motion irrespective of any eccentricities in the superstructure. The question of conservatism, or, alternatively, the adequacy of current practice, needs an extensive scrutiny in light of this phenomenon. In a paper addressed to this question (2), the concept of filtering and the various filter functions associated with arbitrary foundation shapes was introduced. However, the principal results presented there had been restricted to the case of plane SH waves with general oblique incidence for surface foundations and with horizontal incidence for embedded foundations. Recently, the results for embedded discs subjected to obliquely incident body waves and surface waves have been reported by Ray et al. (3). In this paper, the filtering concept is briefly reviewed and applied to the case of generally obliquely incident body waves, namely, SH (or Love waves of layered media), SV, and P waves, and of surface waves of homogeneous half-space, namely, Rayleigh waves, for surface and embedded rectangular foundations. In presenting results for various wave types, dependence of the filter functions on various dimensionless parameters is demonstrated and discussed. Finally, the paper addresses the problem of treating wave-scattering effects for real seismic excitations.

FORMULATION: RIGID FOUNDATION FILTERING

A rigid, massless foundation, welded to and resting on the ground surface or embedded in the earth, forces the media immediately in contact with the foundation to behave as a rigid body. As a result, much of the power associated with high-frequency oscillations of the particles, or, equivalently, with incident waves of wave length comparable with or shorter than the corresponding foundation dimension, is scattered back into the medium; this is what is termed the wave-scattering effect.

The effective translational or rotational input for a foundation is defined here as that motion which would be experienced by a massless, rigid foundation. Assumption of masslessness of the foundation is made to separate the phenomenon of wave scattering from other effects of soil-structure interaction, such as inertial couplings of the superstructure and the media. A complete analysis of soil-structure interactions for a plane incident wave striking a foundation with inertial mass and with its superstructure can be formulated once the effective input of a massless rigid foundation is obtained (4, 5).

Various effective input motions may be obtained by considering that a rigid foundation, resting on the ground surface or embedded in the earth, when subjected to different incident waves through the supporting medium, presents itself as a variety of filters, the nature of which depends on the mode, velocity, and direction of the propagating wave and on the geometry of the contact surface. For example, a rigid foundation resting on the ground surface and struck by a horizontally propagating, horizontally polarized shear (SH) wave or a Love wave will appear as a low-pass horizontal translational and torsional filter about the vertical axis; but it will behave as a low-pass rocking filter about the horizontal axis for obliquely incident compressional and vertically polarized shear (SV) body waves and Rayleigh waves.

Now, let $\vec{x} = \{x_1, x_2, x_3\}^T$ and o define a rectangular cartesian coordinate system with $\vec{e}_i$, $i = 1, 2, 3$ as unit base vectors, as shown in Figure 1, and let the halfspace, H, be defined by $H = \{\vec{x} | (\vec{x}, \vec{e}_3) \leq o\}$, where $(.,.)$ is the usual inner product (or vector dot product) such that:

$$(\vec{x}, \vec{y}) = \sum_{i=1}^{3} x_i y_i$$

Let us consider now a rigid massless embedded rectangular foundation, with the contact surface with surrounding media designated by Γ.

Let $\vec{u}^{i+r}(\vec{x})\ e^{i\omega t}$ be the total free (i.e., without the presence of the foundation) displacement field at any $\vec{x} \varepsilon H$ due to reflection at the free boundary of H of an harmonic, plane incident wave. Let A be the contact area and I^c be the diagonal matrix of order 3 with $I^c{}_{ii}$, i = 1, 2, 3 as the second moment of area of Γ of the foundation about the i^{th} axis at $\vec{x}^c$, the center of the contact area.

Now, let $\vec{u}^c = \{u_1^c, u_2^c, u_3^c\}^T$ be the vector of effective translational motion and $\vec{\Omega}^c = \{\Omega_1^c, \Omega_2^c, \Omega_3^c\}^T$ be the vector of effective rotational motion of the foundation at $\vec{x}_c$; then,

$$\vec{u}^c = \frac{1}{A}\int_{\Gamma} \vec{u}^{i+r}(\vec{x})d\Gamma$$

and

$$\vec{\Omega}^c = [I^c]^{-1}\int_{\Gamma} \vec{u}^{i+r}(\vec{x}) \times (\vec{x} - \vec{x}^c)d\Gamma$$

where x denotes usual vector cross-product.

Similar expressions restricted to the case of only surface foundation and SH wave have been previously reported (6). Before we proceed to present the results of the investigation, some comment on the description of $\vec{u}^{i+r}$, the free field, is in order. Under the plane wave theory, characterization of $\vec{u}^{i+r}$ becomes increasingly complex for SH, P, SV, and Rayleigh waves, in that order. For example, SH waves, unlike P and SV, undergo no mode conversion; P waves, unlike SV, have no critical angle of incidence; the Rayleigh waves, which in reality are generated by the diffraction of the curved wave fronts at the plane half-space boundary (6), become evanescent or inhomogeneous waves under the plane wave theory. The components of $\vec{u}^{i+r}$ for P and SV waves, for example, are shown in Figures 2a and 2b, respectively.

RESULTS

In what follows, without any loss of generality and for simplicity of presentation, the wave normal $\vec{\nu}$ of the incident plane polarized wave in all cases will be assumed to be in the x_1-x_3 plane, i.e., the azimuthal angle θ = 0. Let k_r (= $k \sin\psi$) be the apparent wave number.

For SH waves, the plots for effective foundation motions for several angles of incidence as a function of a dimensionless frequency parameter, $a_o \equiv \frac{k_r L_1}{2}$, for the surface and the embedded rectangular foundations are shown in Figures 3 and 4, respectively. For the embedded foundation, a_o is replaced by an equivalent dimensionless frequency parameter, $a_o^e \equiv \frac{k_r L_e}{2}$, where L_e is equivalent foundation length given by $L_e = L_1(1 + \frac{2e}{L_1} + \frac{2e}{L_2})$. The choice of such equivalent dimensionless frequency parameter essentially scopes out limits of applicability of the filter formulae for various embedment vs. length ratios, the limit of applicability being given approximately by $a_o^e < 4$. The corresponding plots for P, SV, and Rayleigh waves are shown in Figures 5, 6, and 7, respectively.

For horizontally propagating SH waves, translational response of the foundation decreases with increase in frequency; however, the amplitude of torsional motion assumes a maximum for horizontal propagation. Similar reduction in the amplitude of the translational motion (horizontal and vertical) for SV, P, and Rayleigh waves can also be seen from the figures. But one of the most important features of the foundation motion due to Rayleigh waves and horizontally propagating SV waves is that the vertical component of the motion at the edge of the foundation due to rocking about the x_2-axis is about twice the corresponding amplitude of the free-field motion, as shown in Figure 8.

The degree of accuracy of the responses predicted by the present concept is established by comparing those obtained by numerical solution of the associated boundary value problems (5, 8) as shown in Figures 3, 9, and 10. Improved filters that use the impedance characteristics of the half-space for normalization have been derived to predict the foundation responses with an accuracy comparable with those of (5, 8) and will be reported in a separate paper (9). The effect of embedment-lateral dimension ratio on the foundation response is shown in Figure 11.

For the cases in which signal input to the foundation is directionally coherent -- in the sense that the signal consists of only one type of wave, such as one of the surface or body waves propagating through the medium in one particular direction -- numerical evaluation of the response of a massless foundation can be readily carried out through Fourier synthesis. Consider an embedded rectangular foundation, subject to a horizontally polarized shear wave (SH) incident at an angle of 90°, whose corresponding free-field acceleration time history is shown in Figure 12. For this acceleration time history, the effective translational, torsional, and rocking motions obtained through application of the fast Fourier transform (FFT) algorithm (10) for L_1 = 150 ft, embedment e = 15 ft, shear wave velocity V = 2,000 ft/sec, and 7% damping are shown in Figure 13.

In the foregoing discussion we have treated the seismic input in a very simplified way as consisting of plane waves of a single propagation mode and angle of incidence. However, strong motion is usually composed of body waves transmitted directly from the earthquake source, body waves reflected and refracted in the earth's upper layers, and numerous surface wave modes. Also, in the near field of an earthquake, there will be variations in the azimuths of incoming waves. The characteristics of the ground motion depend in detail on the characteristics of the earthquake source, the disposition of the site in relation to the source, and the structure of the upper layers of the earth. Consequently it is not possible to make useful generalizations about the characteristics of seismic strong motion.

If the source-geologic, structure-site configuration is such that near-vertically propagating shear waves dominate the strong motion, then the assumption of horizontally incident shear motion will seriously overestimate the effective torsional input and will underestimate the high-frequency translational input. For example, Newmark's (11) treatment is not applicable to the case of predominantly vertical incidence. Similarly, if obliquely incident body waves contribute significantly to the ground motion, the widely adopted assumption of vertically incident shear waves (12) will overestimate the high-frequency translational motion while overlooking torsion and rocking. It is evident that an assessment of the composition of the strong motion in terms of mode and direction of propagation must precede any reliable soil-structure interaction analysis and, in the present context, must precede the application of the filter functions described above.

To date, only a relatively small volume of work on the problem of the composition of strong motion has appeared in the literature. Given recordings of three components of ground motion at a point, it is an extremely difficult, if not intractable, inverse problem to infer precisely the composition of the motion. However, for the purpose of constructing meaningful "effective" response spectra, decomposition to account for dominant wave types including their directivity can be achieved for "associated" earthquake mechanisms (13).

SUMMARY

An important class of soil-structure interaction phenomena -- wave scattering -- manifested by the attenuation of high-frequency components in the translation motion of the foundation, along with torsional or rocking motion, is discussed. It is shown that the rigid foundation may be visualized as a variety of filters converting the incoming signal into effective foundation motion. The plots for translational, torsional, and rocking filter functions for SH, SV, P, and Rayleigh waves are presented. The effects of such filtering on a typical strong-motion record, with the assumption that it is directionally coherent, are shown in terms of changes in response spectra. The complex nature of the real seismic motion and the need for further research in this area is discussed.

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REFERENCES

1. Yamahara, H., "Ground Motions During Earthquakes and the Input Loss of Earthquake Power to an Excitation of Buildings," Soils and Foundations, The Japanese Society of Soil Mechanics and Foundation Engineering, Vol. X, No. 2, June 1970.

2. Ray, D., and Jhaveri, D.P., "Effective Seismic Input Through Rigid Foundation Filtering," Nuclear Engineering and Design, Vol. 45, No. 1, January 1978, pp. 185-195.

3. Ray, D., et al., "Dynamic Response of Surface and Embedded Disk Foundations for SH, SV, P and Rayleigh Wave Excitations," Proceedings, Sixth Indian Symposium on Earthquake Engineering, Vol. I, India, 1978.

4. Kobori, T., and Shinozaki, Y., "Torsional Vibration of Structures Due to Obliquely Incident SH Waves," Proceedings, Fifth European Conference on Earthquake Engineering, No. 22, Istanbul, 1975.

5. Wong, H.L., and Luco, J.E., "Dynamic Response of Rectangular Foundations to Obliquely Incident Seismic Waves," Earthquake Engineering and Structural Dynamics, Vol. 6, 1978, pp. 3-16.

6. Tani, S., et al., "The Effect of Plane Shape and Size of Buildings on the Input Earthquake Motions," Proceedings, Fifth World Conference on Earthquake Engineering, Vol. 2, Rome, 1974, pp. 1927-1936.

7. Love, A.E.H., "On the Propagation of Tremors over the Surface of an Elastic Solid," Transactions of the Royal Philosophical Society, Ser. A., Vol. 203, 1904, pp. 1-42.

8. Luco, J.E., and Wong, H.L., "Dynamic Response of Rectangular Foundations for Rayleigh Wave Excitation," Proceedings, Sixth World Conference on Earthquake Engineering, New Delhi, January 1977, pp. 4-85 - 4-90.

9. Ray, D., "Improved Rigid Foundation Filtering for Effective Seismic Input" (in preparation).

10. Cooley, J.W., and Tukey, J.W., "An Algorithm for the Machine Calculation of Complex Fourier Series," Mathematics of Computation, Vol. 19, 1965, pp. 297-301.

11. Newmark, N.M., "Comparisons of Building Response and Free Field in Earthquakes," Proceedings, Sixth World Conference on Earthquake Engineering, New Delhi, January 1977.

12. Lysmer, J., Udaka, T., Tsai, C.F., and Seed, H.B., FLUSH -- A Computer Program for Approximate 3-D Analysis of Soil-Structure Interaction Problems, Earthquake Engineering Research Center, Report No. EERC75-30, University of California, Berkeley, 1975.

13. Ray, D., et al., "SECSI -- Seismic Environment Concept for Soil-Structure Interaction" (in preparation).

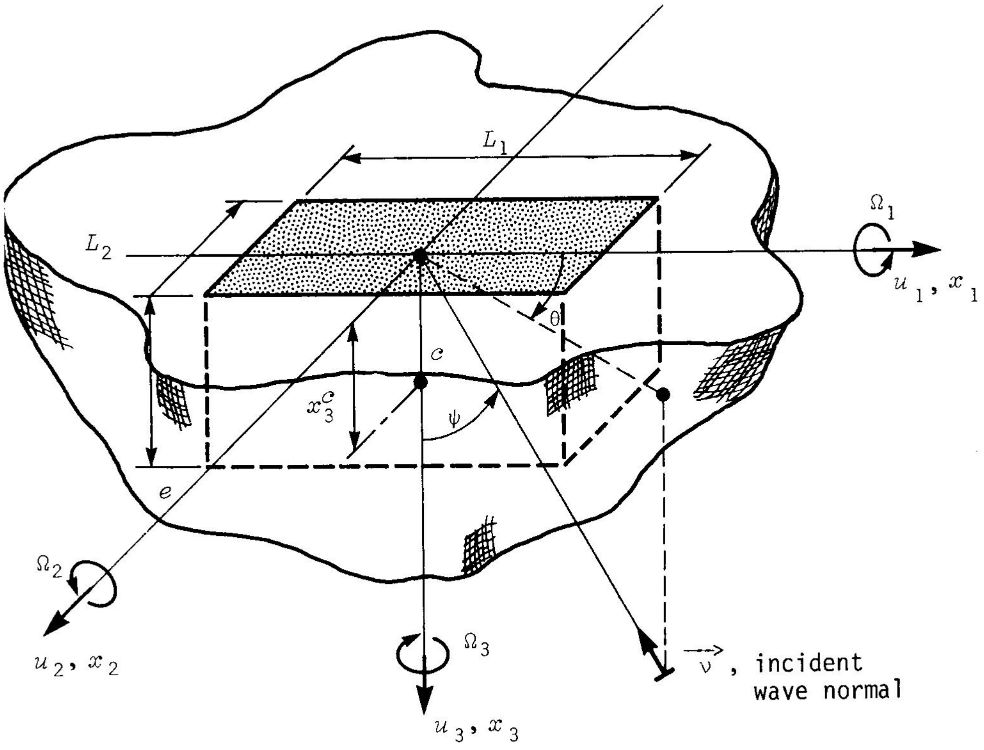

FIGURE 1 BASIC FEATURES OF THE PROBLEM

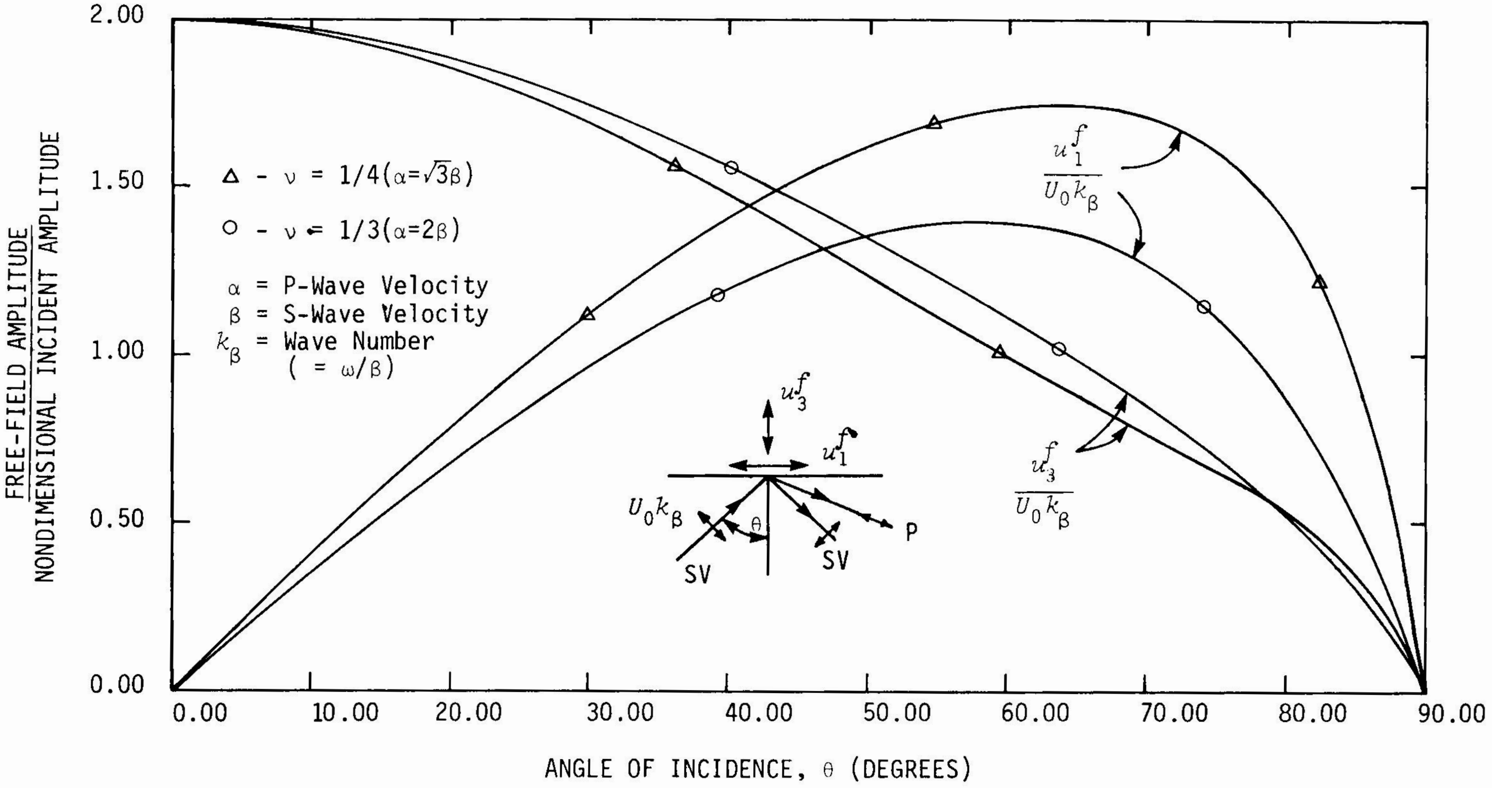

FIGURE 2a FREE-FIELD MOTION VERSUS INCIDENT ANGLE FOR P WAVE

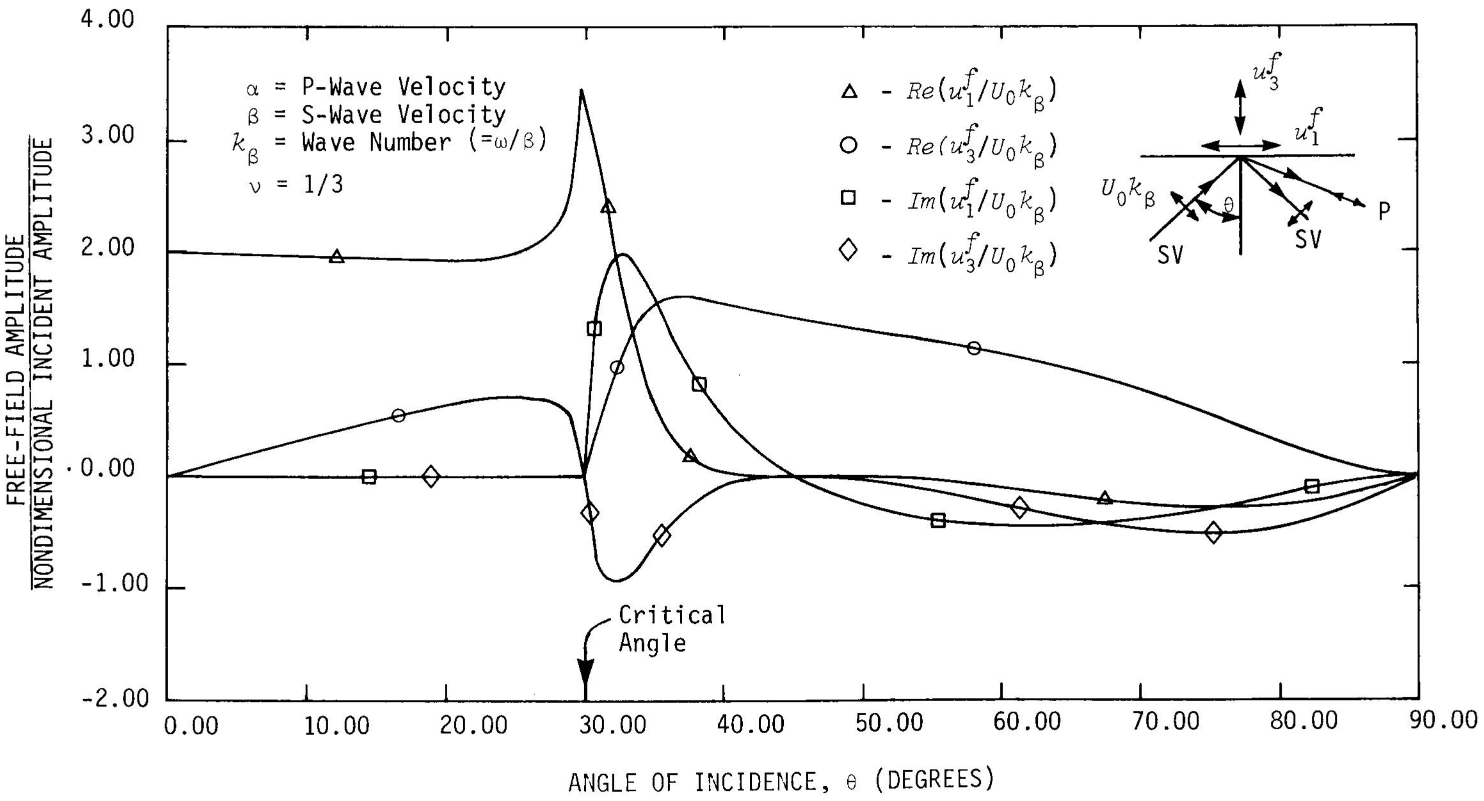

FIGURE 2b FREE-FIELD MOTION VERSUS INCIDENT ANGLE FOR SV WAVE

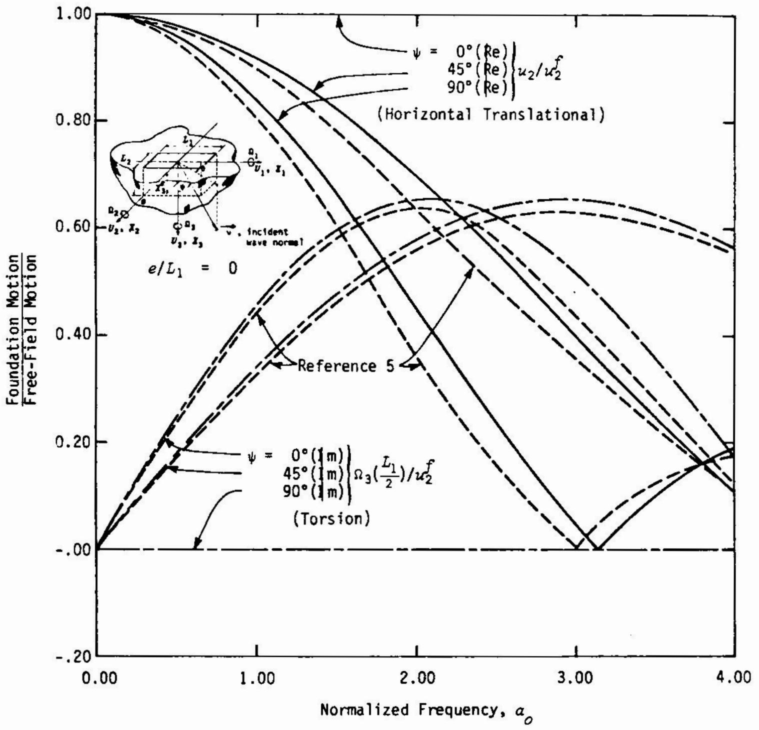

FIGURE 3 SH WAVE - SURFACE FOUNDATION: INPUT MOTION

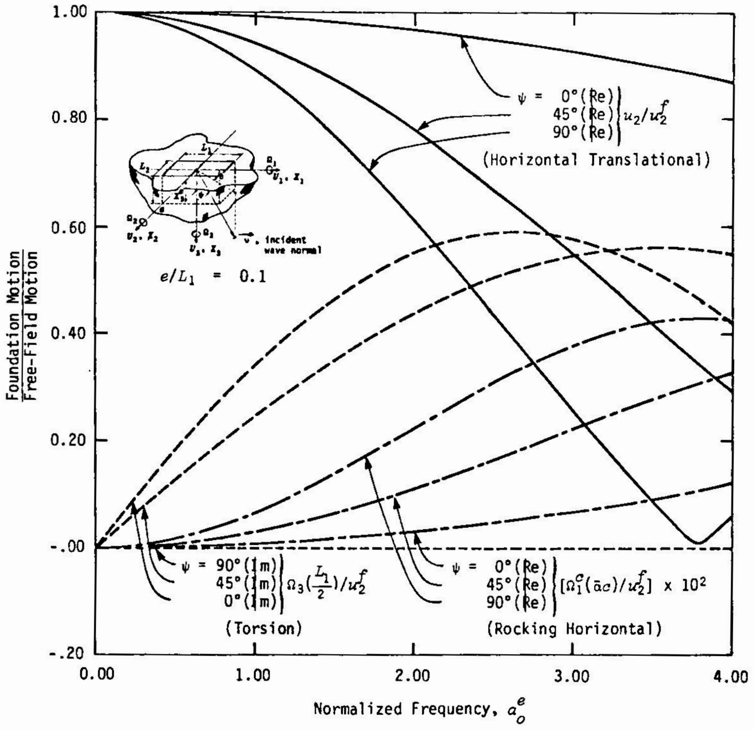

FIGURE 4 SH WAVE - EMBEDDED FOUNDATION: INPUT MOTION

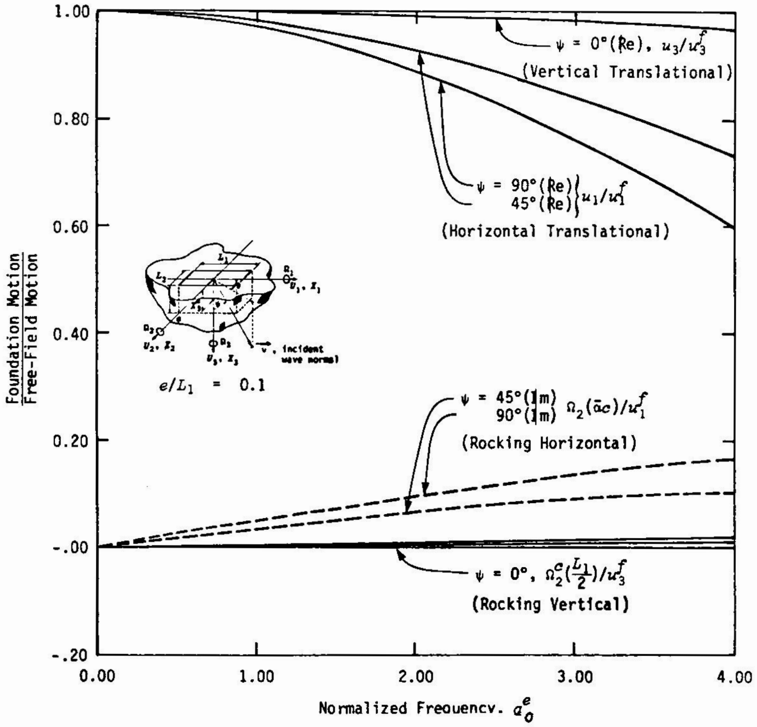

FIGURE 5 P WAVE - EFFECTIVE INPUT MOTIONS

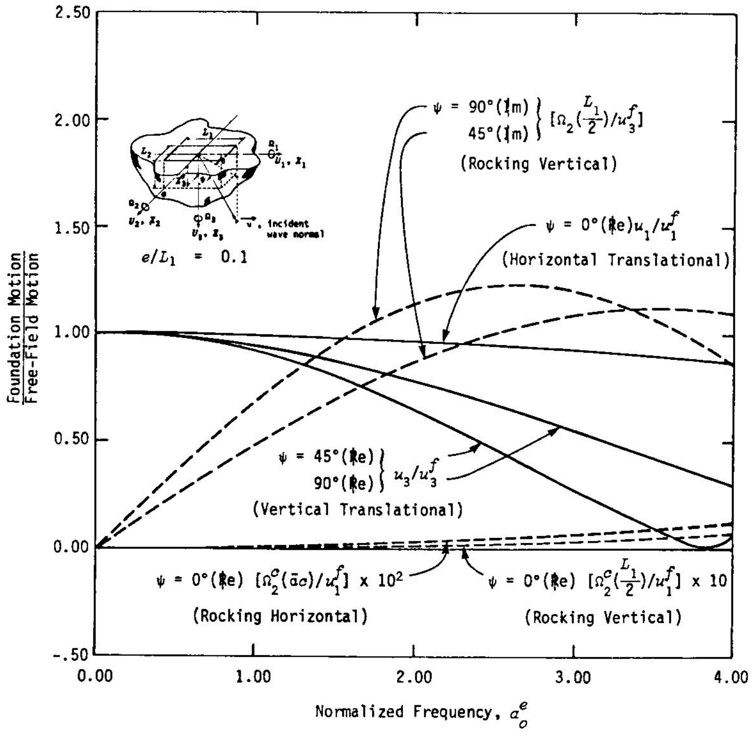

FIGURE 6 SV WAVE - EFFECTIVE INPUT MOTIONS

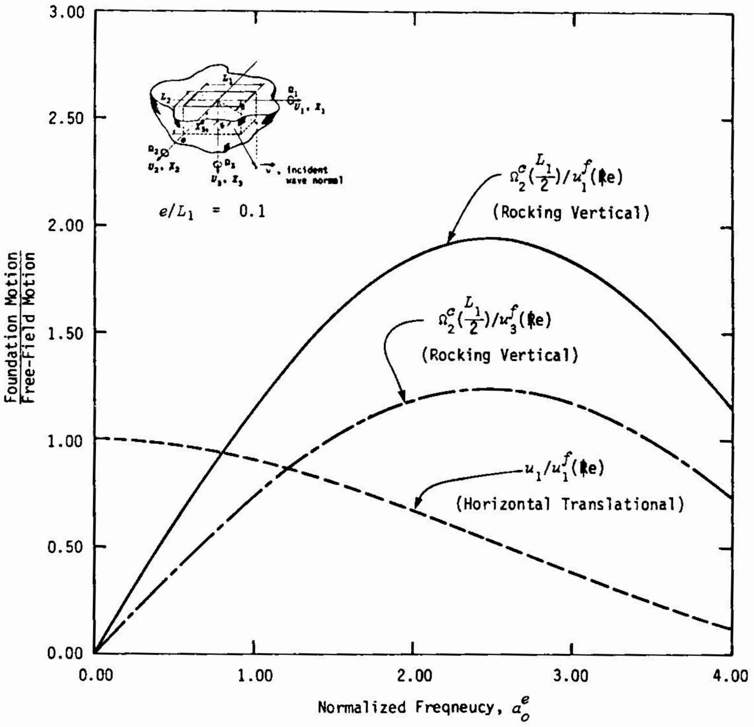

FIGURE 7 RAYLEIGH WAVE - EFFECTIVE INPUT MOTIONS

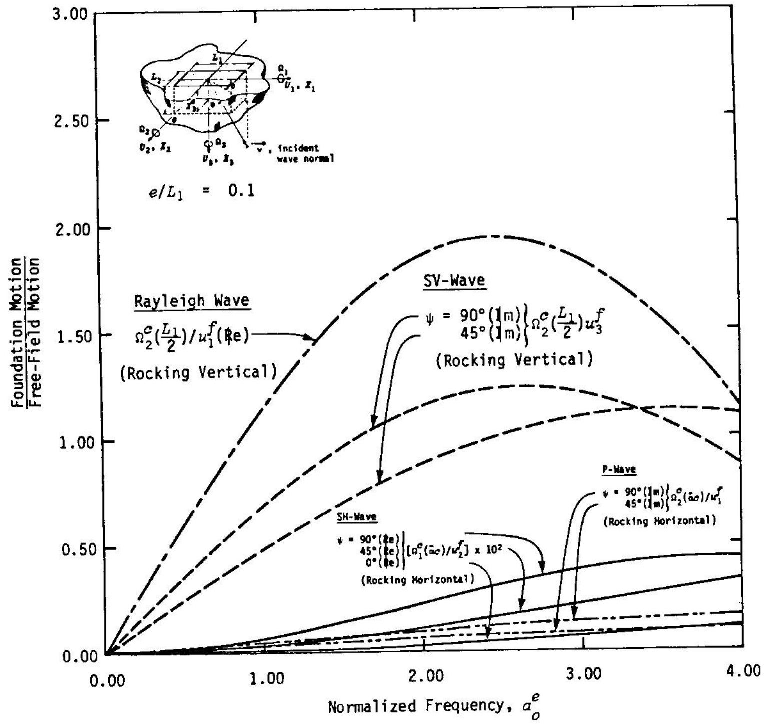

FIGURE 8 COMPARISON OF ROCKING FOR SH, P, SV, AND RAYLEIGH WAVES

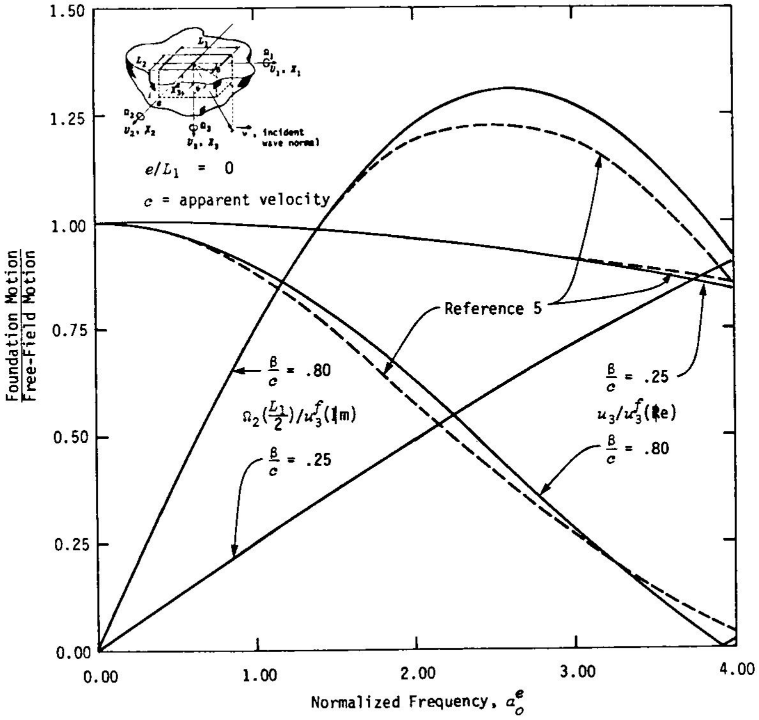

FIGURE 9 SV AND P WAVES - COMPARISON WITH WONG AND LUCO (Reference 5)

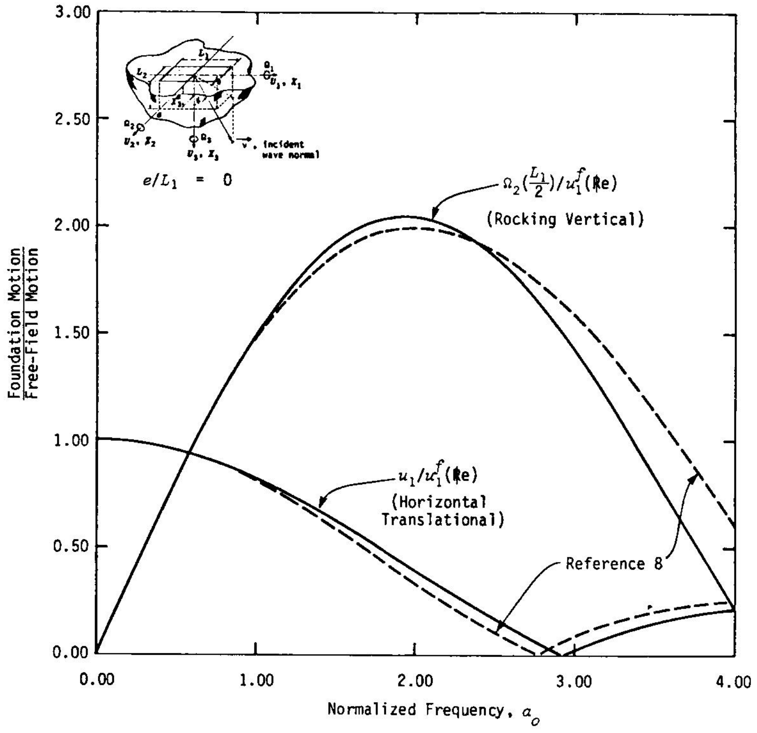

FIGURE 10 RAYLEIGH WAVE - COMPARISON WITH LUCO AND WONG (Reference 8)

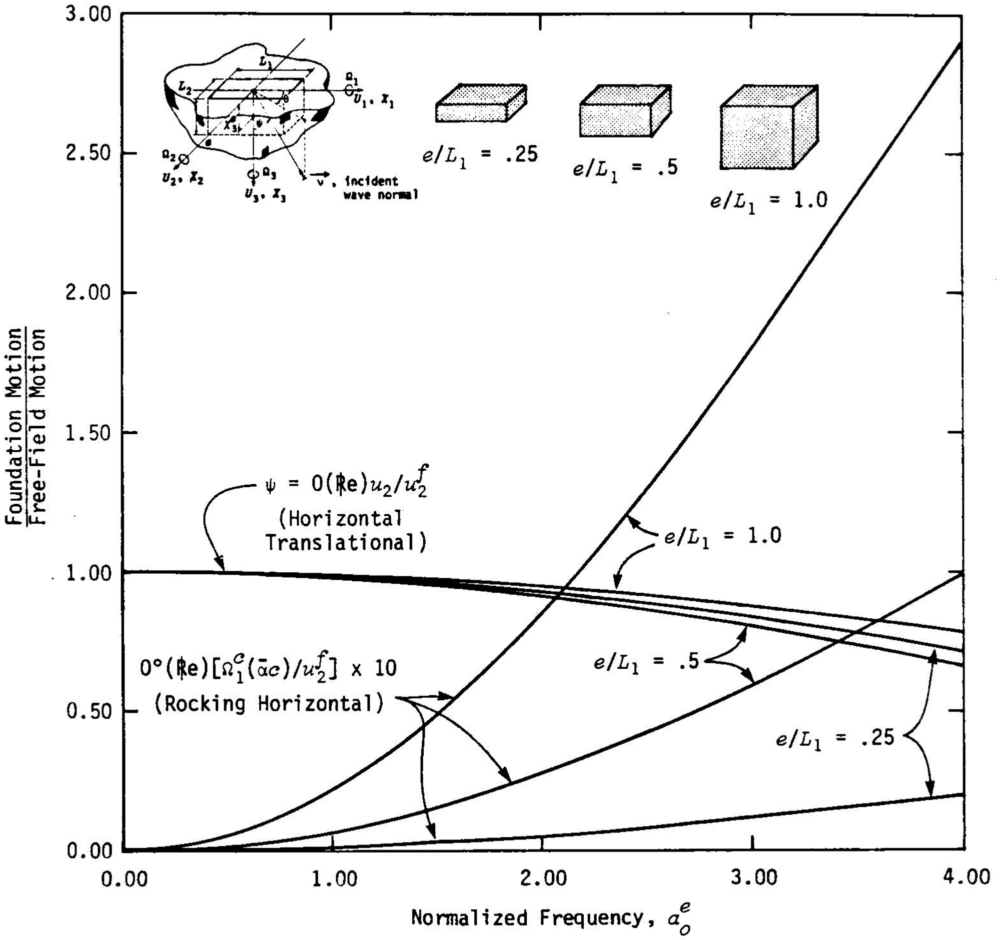

FIGURE 11 SH WAVE - INFLUENCE OF RATIO OF EMBEDMENT TO LATERAL WIDTH

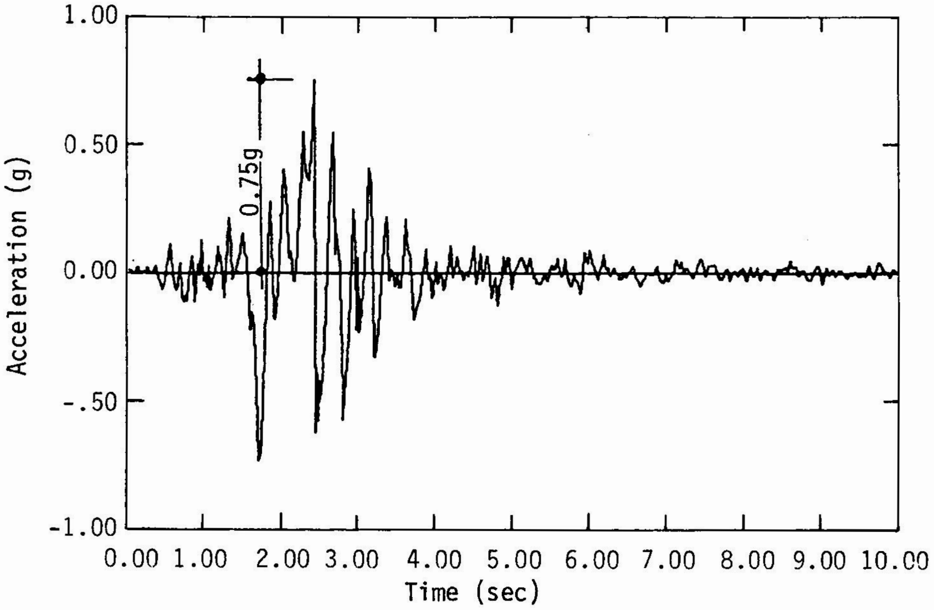

FIGURE 12 ACCELERATION TIME HISTORY

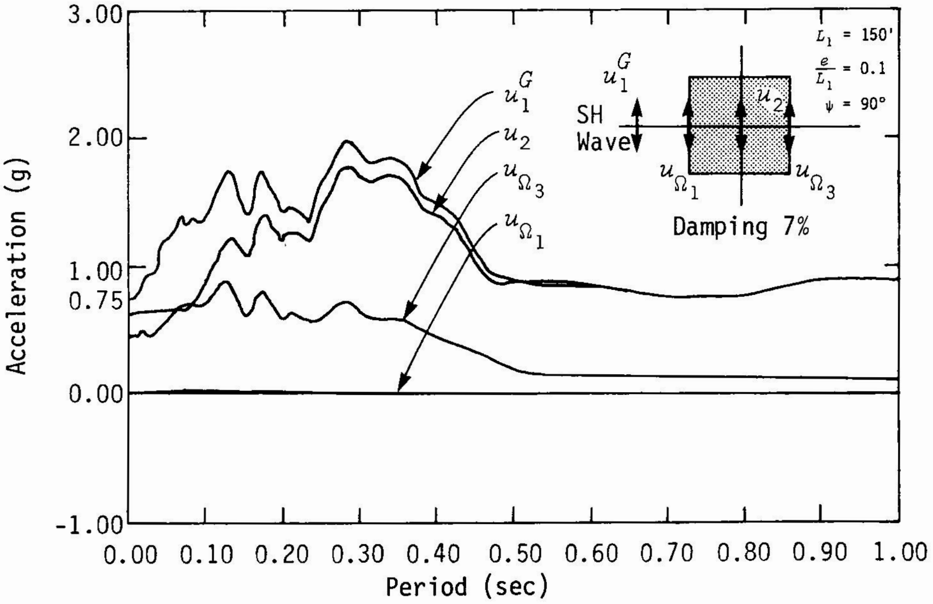

FIGURE 13 FREE-FIELD AND EFFECTIVE RESPONSE SPECTRA